3.4 Systems of Linear Equations

Problems Worksheet

Step Up In Mathematics Specialist Unit 3 and 4

1. Consider the following system of equations.

$$\begin{cases} x - 2y + z = -1 \\ -x + 3y + z = 8 \\ x - y + 4z = 9 \end{cases}$$

   a. Write the system in augmented matrix form.

   b. Use the techniques of elementary row reduction to reduce the system to row echelon form.

   c. Determine the unique solution for the system of equations.

2. Consider the following system of equations.

$$\begin{cases} 4x + y + z = 4 \\ x - y + 2z = 11 \\ 2x + y + z = 4 \end{cases}$$

a. Write the system in augmented matrix form.

b. Use the techniques of elementary row reduction to reduce the system to row echelon form.

c. Determine the unique solution for the system of equations.

3. Consider the following system of equations.

$$\begin{cases} w + x + 2y - z = 6 \\ 2w - 2x + y + z = 4 \\ 3w - x - y - z = 4 \\ 2w + x + 3y + z = 5 \end{cases}$$

Determine the unique solution for this system.

4. Consider the following system of equations.

$$\begin{cases} a + 2b - 2c = -2 \\ 2a + b - c = 2 \\ -2a + 2b + pc = q \end{cases}$$

a. Write the system in augmented matrix form and hence reduce it to row echelon form.

b. Determine the value(s) of p and q so that the system has no solutions.

c. Determine the value(s) of p and q so that the system has infinite solutions.

d. Given that $p = 1$ and $q = -2$, determine the unique solution to this system.

5. Consider the following system of equations.

$$\begin{cases} -2x + y + 3z = -5 \\ x - y - z = 4 \\ -2x + 3y + p^2 z = q \end{cases}$$

a. Write the system in augmented matrix form and hence reduce it to row echelon form.

b. Determine the value(s) of p and q so that the system has no solutions.

c. Determine the value(s) of p and q so that the system has infinite solutions.

6. Consider the three planes defined below.

$\Pi_1: \boldsymbol{r} = \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\Pi_2: \boldsymbol{r}.\begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = c$

$\Pi_3: 2x - 5y + pz = q$

a. Determine any restrictions on the value(s) of c, p and q so that any point on Π_1 is an intersection of the three planes.

b. Determine any restrictions on the value(s) of c, p and q so that there are no points where the three planes intersect.

c. Determine any restrictions on the value(s) of c, p and q so that the planes intersect in a line.

d. Are there possible values or restrictions for c, p and q such that these planes will have one unique point of intersection? Justify your response.